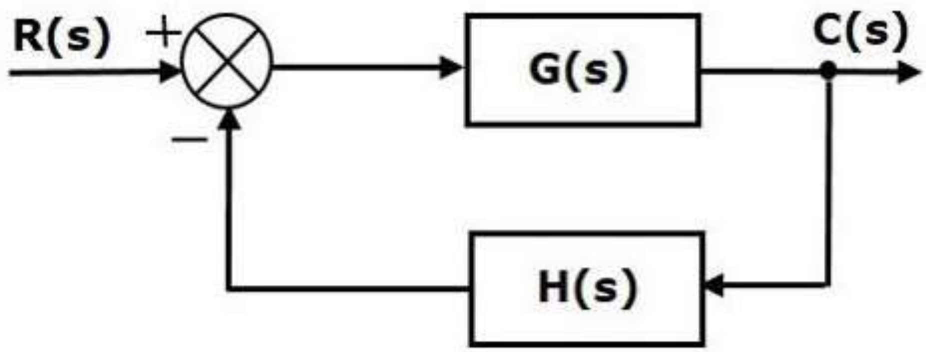
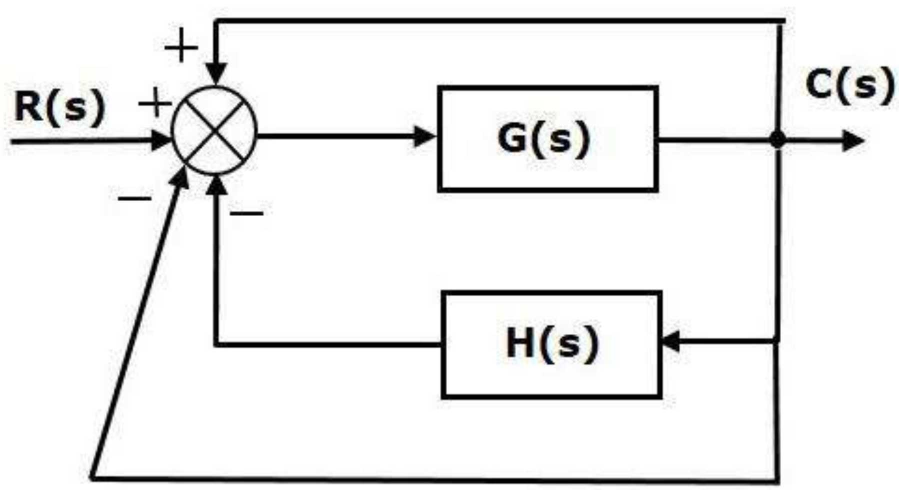
Lecture4: P, I, D, PI, PD and PID Modes of Feedback Control

4.1 Introduction:

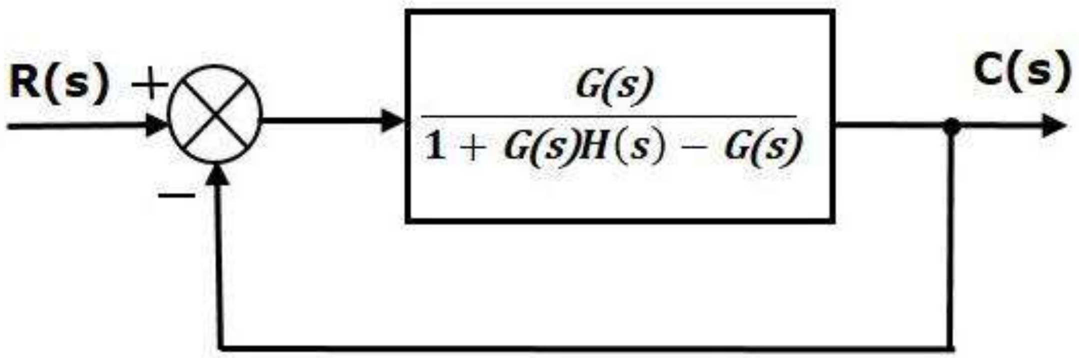
Consider the following block diagram of closed loop control system, which is having non unity negative feedback. Linear control systems use negative feedback to produce a control signal to maintain the controlled **process variable (PV)** at the desired **set point (SP)**.



We can find the steady state errors only for the unity feedback systems (The steady state error is the value of error signal when $t \rightarrow \infty$. The steady state error is the measure of system accuracy. These errors arise form the nature of inputs, type of system and from non-linearity of system components). So, we have to convert the non-unity feedback system into unity feedback system. For this, include one unity positive feedback path and one unity negative feedback path in the above block diagram. The new block diagram looks like as shown below.



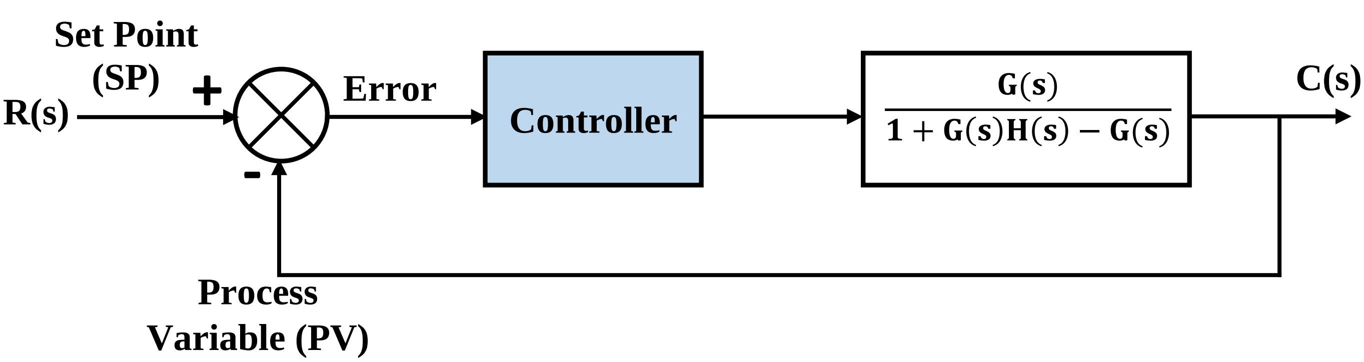
Simplify the above block diagram by keeping the unity negative feedback as it is. The following is the simplified block diagram



This block diagram resembles the block diagram of the unity negative feedback closed loop control system. Here, the single block is having the transfer function G(s) / [1+G(s)H(s)−G(s)] instead of G(s). You can now calculate the steady state errors by using steady state error formula given for the unity negative feedback systems.

4.2 Controllers:

This controller monitors the controlled process variable (PV), and compares it with the reference or set point (SP). The difference between actual and desired value of the process variable, called the **error signal**, or **SP-PV error**, is applied to generate a control action to bring the controlled process variable to the same value as the set point.



4.2.1 Important Advantages of the controller:

The important uses of the controllers are written below:

1. Controllers improve the steady state accuracy by decreasing the steady state error.
2. As the steady state accuracy improves, the stability also improves.
3. Controllers also help in reducing the unwanted offsets produced by the system.
4. Controllers can control the maximum overshoot of the system.
5. Controllers can help in reducing the noise signals produced by the system.
6. Controllers can help to speed up the slow response of an over damped system.

Note: Offset error is the difference between the desired value and the actual value, SP − PV error.

4.2.2 Types of Controller:

There are two main types of controllers: **continuous controllers**, and **discontinuous controllers**.

In **discontinuous controllers**, the manipulated variable (controlled variable) changes between discrete values. Depending on how many different states the manipulated variable can assume, a distinction is made between two-position, three-position and multi-position controllers. Compared to continuous controllers, discontinuous controllers operate on very simple, switching final controlling elements.

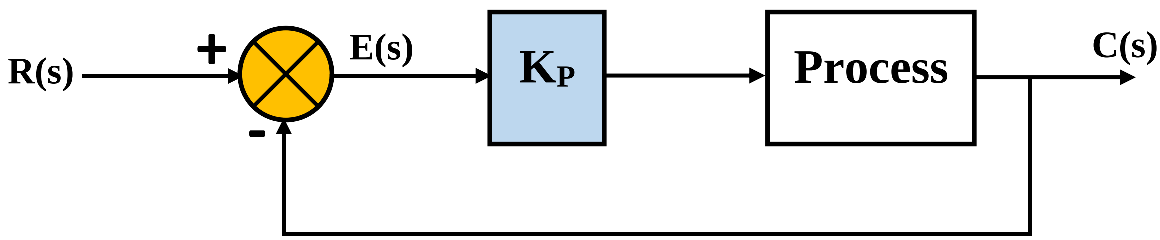
The main feature of **continuous controllers** is that the controlled variable (also known as the manipulated variable) can have any value within controller’s output range.

Note: For sequential and combinational logic, software logic, such as in a programmable logic controller (PLC), is used. Today, most such systems are constructed with microcontrollers or more specialized programmable logic controllers (PLCs).

Now in the continuous controller theory, there are various types of controllers used to improve the performance of control systems.

4.2.2.1 Proportional Controller:

For a controller with proportional control action, the relationship between the output of the controller **c(t)** and the actuating error signal **e(t)** is:



$$c(t) = K_P\, e(t)$$

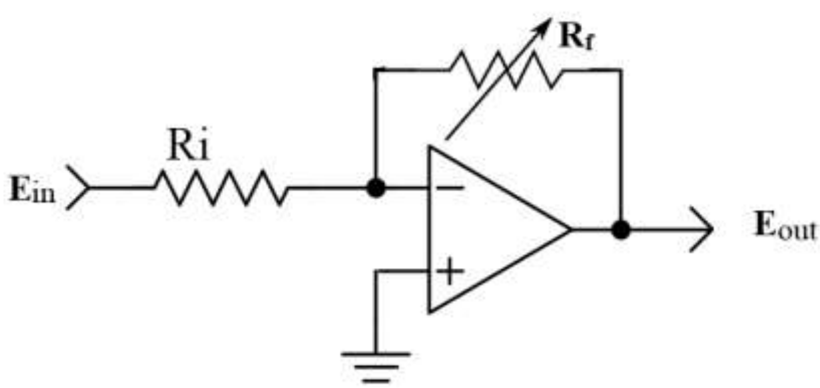
or, in Laplace-transformed quantities,

$$\frac{C(s)}{E(s)} = K_P$$

where $\mathbf{K_P}$ is termed the proportional gain.

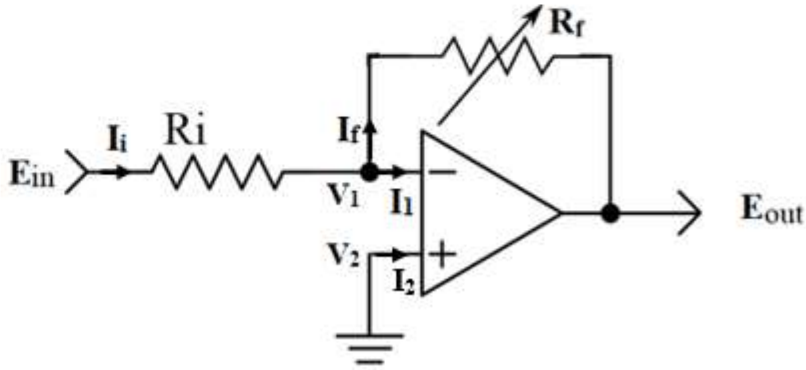
Whatever the actual mechanism may be and whatever the form of the operating power, the proportional controller is essentially an amplifier with an adjustable gain.

Example 1: Find the transfer function for the controller circuit shown in figure below. Then, write the type of the controller.



Solution:

For ideal operational amplifier, the currents flow into the input terminals are zero ($I_1 = I_2 = 0$) and the differential input voltage is zero ($E_{out} = v_2 - v_1 \rightarrow v_1 = v_2 = 0$). where $Z_1 = R_i$ and $Z_2 = R_f$.



$$I_i = I_1 + I_f \rightarrow I_i = I_f$$

$$\frac{E_{in} - v_1}{Z_1} = \frac{v_1 - E_{out}}{Z_2}$$

$$\frac{E_{in} - 0}{Z_1} = \frac{0 - E_{out}}{Z_2}$$

$$\therefore \mathrm{T.F.} = \frac{E_{out}}{E_{in}} = -\frac{Z_2}{Z_1} = -\frac{R_f}{R_i}$$

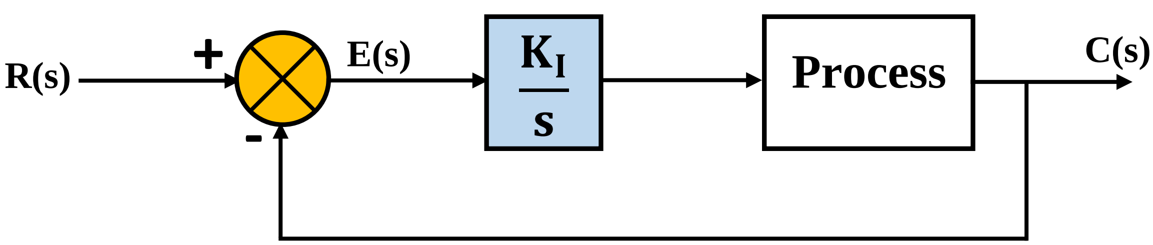
This controller is proportional controller.

Since $\frac{C(s)}{E(s)} = K_P$

$$\therefore K_P = -\frac{R_f}{R_i}$$

4.2.2.2 Integral Controller:

In a controller with integral control action, the value of the controller output **c(t)** is changed at a rate proportional to the actuating error signal **e(t)**. That is,



$$\frac{dc(t)}{dt} = K_I \, e(t)$$

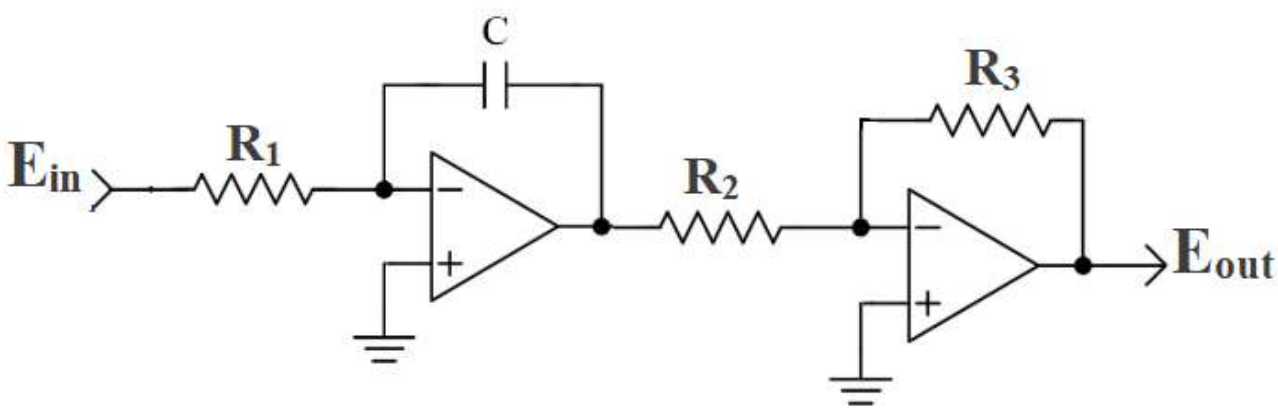
or

$$c(t) = K_I \int_0^t e(t)\, dt$$

where **K_i** is an adjustable constant. The transfer function of the integral controller is:

$$\frac{C(s)}{E(s)} = \frac{K_I}{s}$$

Example 2: Find the transfer function for the controller circuit shown in figure below. Then, write the type of the controller.



Solution:

For ideal both operational amplifiers, the currents flow into the input terminals are zero ($I_1 = I_2 = 0$) and the differential input voltage is zero ($E_{out} = v_2 - v_1 \rightarrow v_1 = v_2 = 0$).

where $Z_1 = R_1$, $Z_2 = X_C = \frac{1}{j\omega C} = \frac{1}{sC}$, $Z_3 = R_2$, and $Z_4 = R_3$

$$\text{T. F of circuit1 (Integrator op} - \text{amp)} = \frac{E_{out1}}{E_{in}}$$

$$\text{T. F of circuit2 (Inverter op} - \text{amp)} = \frac{E_{out}}{E_{out1}}$$

Then,

$$\text{T. F of the total circuit (both op} - \text{amp)} = \frac{E_{out1}}{E_{in}} \times \frac{E_{out}}{E_{out1}} = \frac{E_{out}}{E_{in}}$$

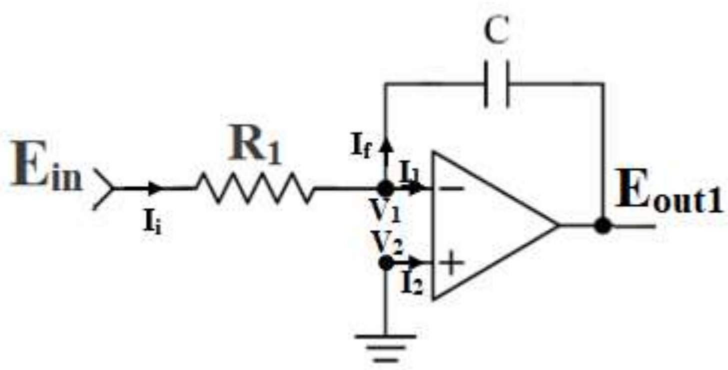
For part1 of the circuit (Integrator op − amp):

$I_i = I_1 + I_f \rightarrow I_i = I_f$

$$\frac{E_{in} - v_1}{Z_1} = \frac{v_1 - E_{out1}}{Z_2}$$

$$\frac{E_{in} - 0}{Z_1} = \frac{0 - E_{out1}}{Z_2}$$

$$\text{T. F.} = \frac{E_{out1}}{E_{in}} = -\frac{Z_2}{Z_1} = -\frac{\frac{1}{sC}}{R_1}$$



$$\therefore \text{T.F.} = -\frac{1}{sC} \times \frac{1}{R_1} = -\frac{1}{sCR_1}$$

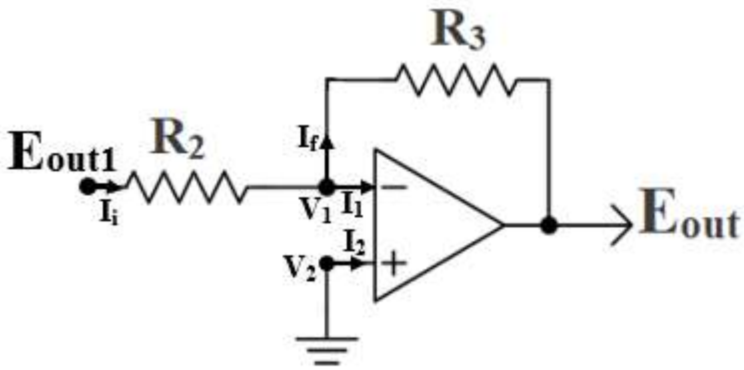
For part 2 of the circuit (Inverting op − amp):

$I_i = I_1 + I_f \rightarrow I_i = I_f$

$$\frac{E_{out1} - v_1}{Z_3} = \frac{v_1 - E_{out}}{Z_4}$$

$$\frac{E_{out1} - 0}{Z_3} = \frac{0 - E_{out}}{Z_4}$$

$$\therefore \text{T.F.} = \frac{E_{out}}{E_{out1}} = -\frac{Z_4}{Z_3} = -\frac{R_3}{R_2}$$



$$\text{T.F of the controller circuit (both op − amps)} = \frac{E_{out1}}{E_{in}} \times \frac{E_{out}}{E_{out1}} = \frac{E_{out}}{E_{in}}$$

$$= -\frac{1}{sCR_1} \times -\frac{R_3}{R_2} = \frac{R_3}{sCR_1R_2}$$

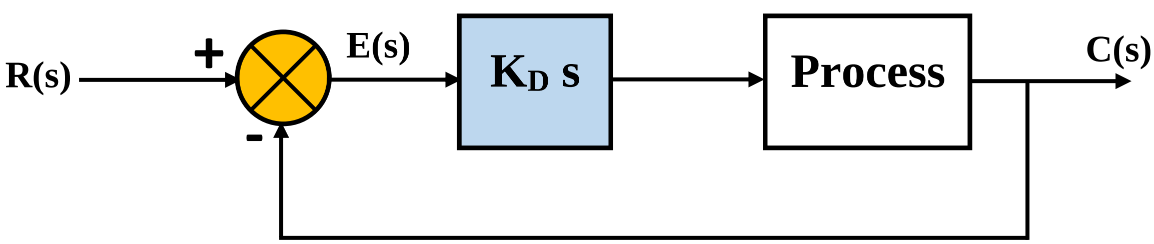
This controller is integral controller.

Since $\frac{C(s)}{E(s)} = \frac{K_I}{s}$

$$\therefore K_I = \frac{R_3}{CR_1R_2}$$

4.2.2.3 Derivative Controller:

The derivative controller produces an output, which is derivative of the error signal.



$$c(t) = K_D \frac{d\,e(t)}{dt}$$

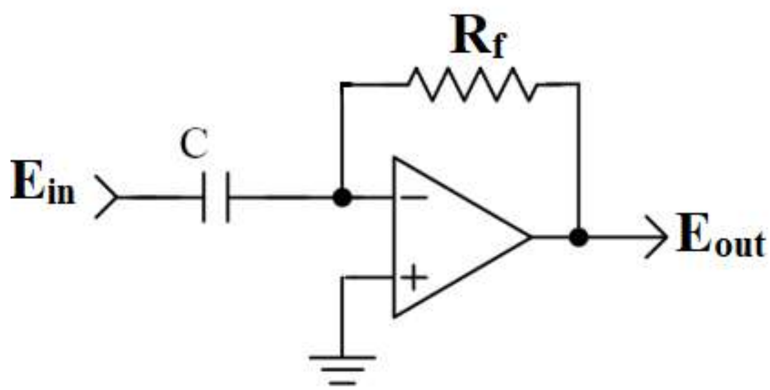
Apply Laplace Transform on both sides.

$$C(s) = K_D\, s\, E(s)$$

$$\frac{C(s)}{E(s)} = K_D\, s$$

Where, $\mathbf{K_D}$ is the derivative constant.

Example 3: Find the transfer function for the controller circuit shown in figure below. Then, write the type of the controller.



Solution:

For ideal operational amplifier, the currents flow into the input terminals are zero ($I_1 = I_2 = 0$) and the differential input voltage is zero ($E_{out} = v_2 - v_1 \rightarrow v_1 = v_2 = 0$).

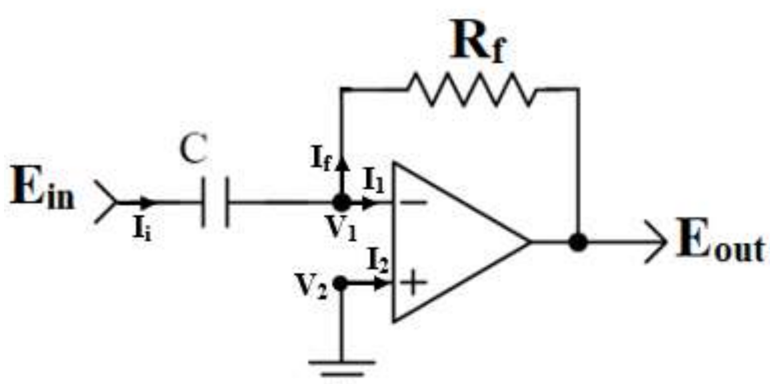
where $Z_1 = X_C = \frac{1}{j\omega C} = \frac{1}{sC}$, and $Z_2 = R_f$

$$I_i = I_1 + I_f \rightarrow I_i = I_f$$

$$\frac{E_{in} - v_1}{Z_1} = \frac{v_1 - E_{out}}{Z_2}$$

$$\frac{E_{in} - 0}{Z_1} = \frac{0 - E_{out}}{Z_2}$$

$$\therefore \text{T. F.} = \frac{E_{out}}{E_{in}} = -\frac{Z_2}{Z_1} = -\frac{R_f}{\frac{1}{sC}} = -sR_fC$$



This controller is derivative controller.

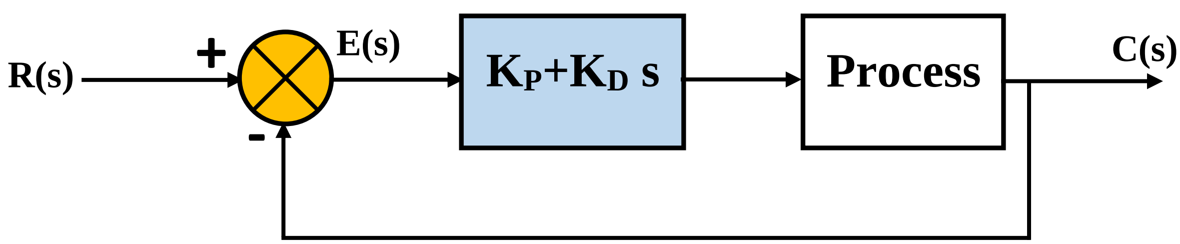
Since

$$\frac{C(s)}{E(s)} = K_D\, s$$

$K_D = -R_f C$

4.2.2.4 Proportional Derivative (PD) Controller:

The proportional derivative controller produces an output, which is the combination of the outputs of proportional and derivative controllers.



$$c(t) = K_P\, e(t) + K_D \frac{d\, e(t)}{dt}$$

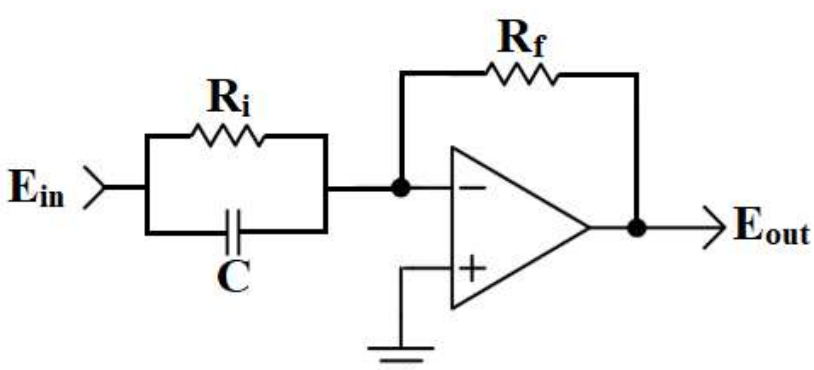
Apply Laplace Transform on both sides.

$$C(s) = K_P\, E(s) + K_D\, s\, E(s)$$

Therefore, the transfer function of the proportional derivative controller is:

$$\frac{C(s)}{E(s)} = K_P + K_D\, s$$

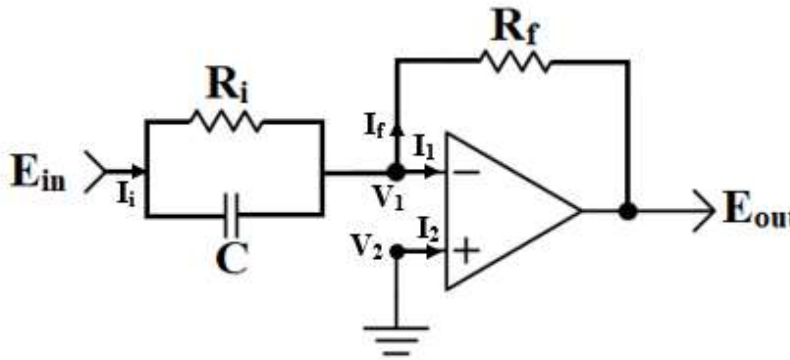
Example 4: Find the transfer function for the controller circuit shown in figure below. Then, write the type of the controller.



Solution:

For ideal operational amplifier, the currents flow into the input terminals are zero ($I_1 = I_2 = 0$) and the differential input voltage is zero ($E_{out} = v_2 - v_1 \rightarrow v_1 = v_2 = 0$).

Where



$$Z_1 = R_i||X_C = \frac{R_i \times \frac{1}{j\omega C}}{R_i + \frac{1}{j\omega C}} = \frac{\frac{R_i}{j\omega C}}{\frac{1+j\omega CR_i}{j\omega C}}$$

$$\therefore\ z_1 = \frac{R_i}{1+j\omega CR_i} = \frac{R_i}{1+sCR_i}, \text{and } Z_2 = R_f$$

$$I_i = I_1 + I_f \rightarrow I_i = I_f$$

$$\frac{E_{in} - v_1}{Z_1} = \frac{v_1 - E_{out}}{Z_2}$$

$$\frac{E_{in} - 0}{Z_1} = \frac{0 - E_{out}}{Z_2}$$

$$\therefore \text{T.F.} = \frac{E_{out}}{E_{in}} = -\frac{Z_2}{Z_1} = -\frac{R_f}{\frac{R_i}{1+sCR_i}} = -\frac{R_f(1+sCR_i)}{R_i} = -(\frac{R_f}{R_i} + sCR_f)$$

This controller is Proportional Derivative (PD) controller.

Since

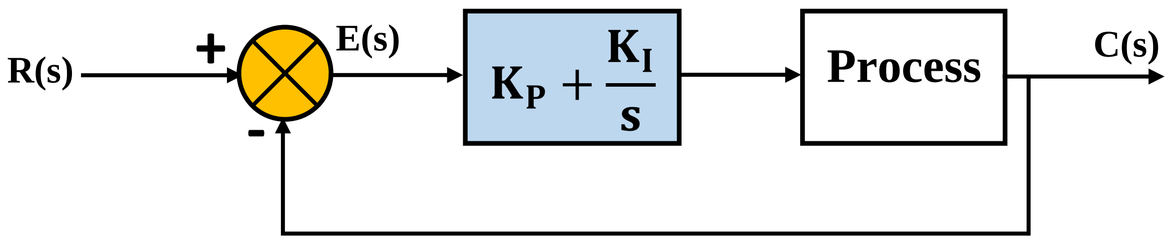
$$\frac{C(s)}{E(s)} = K_P + K_D\, s$$

$$K_P = -\frac{R_f}{R_i}$$

$$K_D = -sCR_f$$

4.2.2.5 Proportional Integral (PI) Controller:

The proportional integral controller produces an output, which is the combination of outputs of the proportional and integral controllers.



$$c(t) = K_P\, e(t) + K_I \int_0^t e(t)\, dt$$

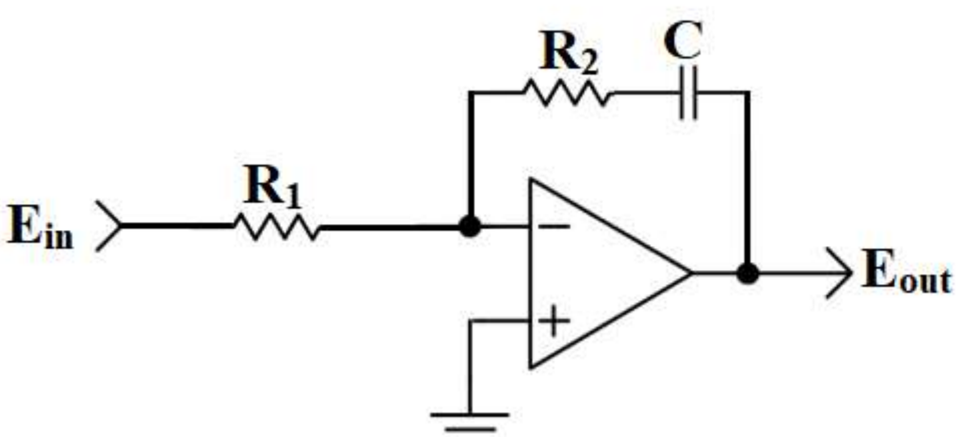
Apply Laplace Transform on both sides.

$$C(s) = K_P\, E(s) + \frac{K_I}{s}\, E(s)$$

Therefore, the transfer function of the proportional integral controller is:

$$\frac{C(s)}{E(s)} = K_P + \frac{K_I}{s}$$

Example 5: Find the transfer function for the controller circuit shown in figure below. Then, write the type of the controller.



Solution:

For ideal operational amplifier, the currents flow into the input terminals are zero ($I_1 = I_2 = 0$) and the differential input voltage is zero ($E_{out} = v_2 - v_1 \rightarrow v_1 = v_2 = 0$).

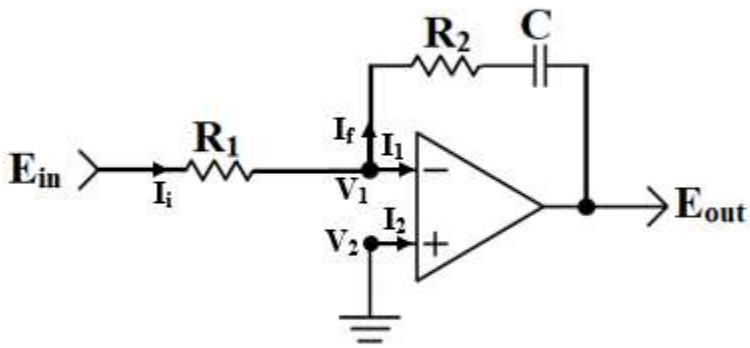
Where

$$z_1 = R_1, \text{and } Z_2 = R_2 + X_C = R_2 + \frac{1}{j\omega C} = \frac{1 + j\omega C R_2}{j\omega C} = \frac{1 + sCR_2}{sC}$$

$I_i = I_1 + I_f \rightarrow I_i = I_f$

$$\frac{E_{in} - v_1}{Z_1} = \frac{v_1 - E_{out}}{Z_2}$$

$$\frac{E_{in} - 0}{Z_1} = \frac{0 - E_{out}}{Z_2}$$



$$\therefore \text{T.F.} = \frac{E_{out}}{E_{in}} = -\frac{Z_2}{Z_1} = -\frac{\frac{1 + sCR_2}{sC}}{R_1}$$

$$= -\frac{(1 + sCR_2)}{sCR_1}$$

$$= -(\frac{1}{sCR_1} + \frac{R_2}{R_1})$$

This controller is Proportional Integral (PI) Controller.

Sine

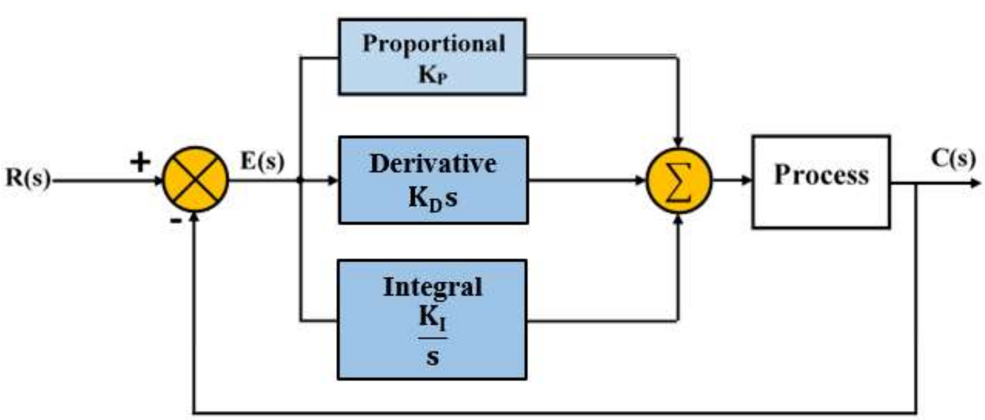
$$\frac{C(s)}{E(s)} = K_P + \frac{K_I}{s}$$

$$K_P = -\frac{R_2}{R_1}$$

$$K_I = -\frac{1}{CR_1}$$

4.2.2.6 Proportional Integral Derivative (PID) Controller:

The proportional integral derivative controller produces an output, which is the combination of the outputs of proportional, integral and derivative controllers.



$$c(t) = K_P\, e(t) + K_I \int_0^t e(t)\, dt + K_D \frac{d\, e(t)}{dt}$$

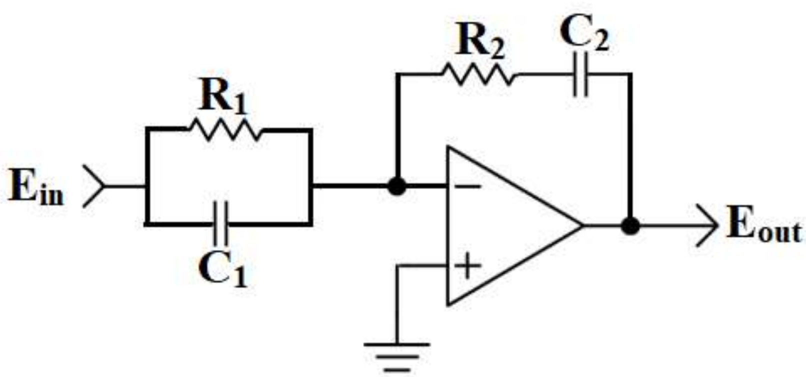
Apply Laplace Transform on both sides.

$$C(s) = K_P\, E(s) + \frac{K_I}{s}\, E(s) + K_D\, s\, E(s)$$

Therefore, the transfer function of the proportional integral derivative controller is:

$$\frac{C(s)}{E(s)} = K_P + \frac{K_I}{s} + K_D\, s$$

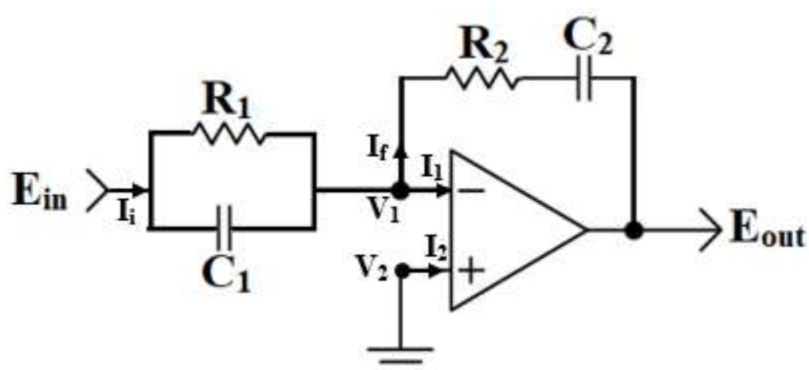
Example 6: Find the transfer function for the controller circuit shown in figure below. Then, write the type of the controller.



Solution:

For ideal operational amplifier, the currents flow into the input terminals are zero ($I_1 = I_2 = 0$) and the differential input voltage is zero ($E_{out} = v_2 - v_1 \rightarrow v_1 = v_2 = 0$).

Where



$$Z_1 = R_1 || X_{C_1} = \frac{R_1 \times \frac{1}{j\omega C_1}}{R_1 + \frac{1}{j\omega C_1}} = \frac{\frac{R_1}{j\omega C_1}}{\frac{1 + j\omega C_1 R_1}{j\omega C_1}}$$

$$z_1 = \frac{R_1}{1 + j\omega C_1 R_1} = \frac{R_1}{1 + sC_1 R_1}$$

$$Z_2 = R_2 + X_{C_2} = R_2 + \frac{1}{j\omega C_2} = \frac{1 + j\omega C_2 R_2}{j\omega C_2} = \frac{1 + sC_2 R_2}{sC_2}$$

$I_i = I_1 + I_f \rightarrow I_i = I_f$

$$\frac{E_{in} - v_1}{Z_1} = \frac{v_1 - E_{out}}{Z_2}$$

$$\frac{E_{in} - 0}{Z_1} = \frac{0 - E_{out}}{Z_2}$$

$$\therefore \mathrm{T.F.} = \frac{E_{out}}{E_{in}} = -\frac{Z_2}{Z_1} = -\frac{\dfrac{1 + sC_2R_2}{sC_2}}{\dfrac{R_1}{1 + sC_1R_1}} = -\frac{1 + sC_2R_2}{sC_2} \times \frac{1 + sC_1R_1}{R_1}$$

$$= -\left(\frac{1 + sC_1R_1 + sC_2R_2 + s^2C_1C_2R_1R_2}{sC_2R_1}\right) = -\left(\frac{C_1R_1 + C_2R_2}{C_2R_1} + \frac{1}{sC_2R_1} + sC_1R_1\right)$$

This controller is Proportional Integral Derivative (PID) Controller.

Sine

$$\frac{C(s)}{E(s)} = K_P + \frac{K_I}{s} + K_D\, s$$

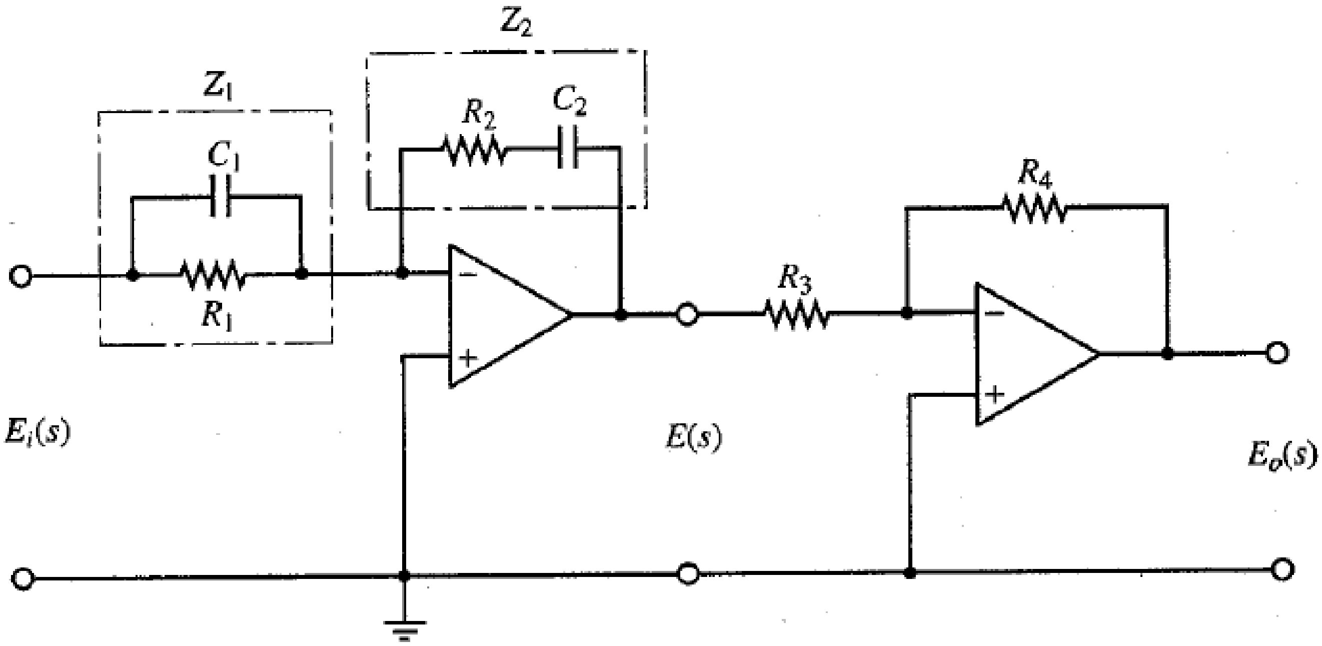
$$K_P = -\frac{C_1R_1 + C_2R_2}{C_2R_1}$$

$$K_I = -\frac{1}{C_2R_1}$$

$$K_D = C_1R_1$$

Note: The by far most used control method in industry is the PID controller. It is currently claimed that 90 to 95% of industrial problems can be solved by this type of controller, which is easily available as an electronic module.

Homework: Find the transfer function for the controller circuit shown in figure below. Then, write the type of the controller.



Solution:

$$\mathrm{T.F.} = \frac{\mathrm{E_o}(s)}{\mathrm{E_i}(s)} = \frac{R_4(R_1C_1 + R_2C_2)}{R_3R_1C_2} + \frac{R_4}{sR_3R_1C_2} + s\,\frac{R_4R_2C_1}{R_3}$$

This controller is Proportional Integral Derivative (PID) Controller.

Sine

$$\frac{C(s)}{E(s)} = K_p + \frac{K_i}{s} + K_D\,s$$

$$K_P = \frac{R_4(R_1C_1 + R_2C_2)}{R_3R_1C_2}$$

$$K_I = \frac{R_4}{R_3R_1C_2}$$

$$K_D = \frac{R_4R_2C_1}{R_3}$$